Logarithmic correction to scaling in domain-wall dynamics at Kosterlitz-Thouless phase transitions

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With Monte Carlo simulations, we investigate the relaxation dynamics of domain walls at the Kosterlitz-Thouless phase transition, taking the two-dimensional *XY* model as an example. The dynamic scaling behavior is carefully analyzed, and a domain-wall roughening process is observed. Two-time correlation functions are calculated and aging phenomena are investigated. Inside the domain interface, a strong logarithmic correction to scaling is detected.

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I. INTRODUCTION

In the past years much progress has been achieved in the study of dynamic processes far from equilibrium. For example, the universal dynamic scaling form in critical dynamics has been explored up to the *macroscopic* short-time regime [1–9], when the system is still far from equilibrium. Although the spatial correlation length is still short in the beginning of the time evolution, the dynamic scaling form is induced by the divergent correlating time around a continuous phase transition. Based on the short-time dynamic scaling form, new methods for the determination of both dynamic and static critical exponents as well as the critical temperature have been developed [7–10]. Since the measurements are carried out in the short-time regime, one does not suffer from critical slowing down. Recent progress in the short-time critical dynamics includes, for example, theoretical calculations and numerical simulations of the *XY* models and Josephson junction arrays [11–14], magnets with quenched disorder [15–19], aging phenomena [20–24], weak first-order phase transitions [16,25–27], and various applications and developments [28–34].

To understand the critical dynamics far from equilibrium, one should keep in mind that the dynamic scaling form is dependent on the *macroscopic* initial condition [1,8,35]. Up to now, the dynamic relaxation with *ordered* and *random* initial states has been systematically investigated. The magnetization decays by a power law for the ordered initial state [7,8,35], yet shows *an initial increase* in the macroscopic short-time regime for the random initial state with a small initial magnetization. An independent critical exponent x_0 must be introduced to describe the scaling dimension of the initial magnetization [1,6,8,9].

On the other hand, many recent activities have been devoted to the domain-wall dynamics. For magnetic materials, a domain wall separates domains with different spin orientations. The magnetic domain-wall dynamics is an important topic in magnetic devices, nanomaterials, and semiconductors [36–43]. For a magnetic system with weak disorder at zero temperature, the domain wall does not propagate unless the external magnetic field h exceeds a threshold h_c. This is the so-called pinning-depinning phase transition. At the critical field h_c, a roughening phenomenon is also observed. When a periodic external field $h(t)=h_0\cos(\omega t)$ is applied and/or a nonzero temperature is introduced, the domain wall exhibits different states of motion and dynamic phase transitions [36,37,42–47]. Most of these works concentrate on the stationary state at the zero or low temperatures and in response to the external magnetic field $h(t)$.

Very recently, the dynamic relaxation of a domain wall has been concerned for magnetic systems at a standard order-disorder phase transition [34,48,49]. It is described by the relaxation dynamics starting from a *semiordered* state, and shares certain common features with those around free and disordered surfaces. Since no external magnetic fields are added, macroscopically the domain wall does not move, but a kind of roughening phenomenon occurs. Furthermore, similar dynamic approaches can be applied to the pinning-depinning and other dynamic phase transitions of domain walls at zero or low temperatures [50], to understand the nonstationary properties of the dynamic systems and determining the static and dynamic exponents as well as the transition points.

In this paper, we investigate the relaxation dynamics of domain walls at a Kosterlitz-Thouless (KT) phase transition, taking the two-dimensional (2D) *XY* model as an example. It is known that the KT phase transition is topological, and topological excitations such as the free vortices and vortex pairs above and below the transition temperature play essential roles. Especially, logarithmic corrections to scaling emerge, not only in equilibrium but also in the nonequilibrium relaxation processes [51]. Determination of the logarithmic correction to scaling is theoretically important, but practically notorious in numerical simulations. Corrections to scaling generally depend on macroscopic initial conditions. For example, the logarithmic correction exists in the dynamic relaxation starting from a disordered state, but is suppressed in the dynamic relaxation starting from an ordered state [11,24,52–54]. In this paper, we aim at understanding the dynamic scaling form and possible corrections to scaling in the dynamic relaxation of a domain wall.

In Sec. II, the model and scaling analysis are described, and in Sec. III, the numerical results are presented. Section IV includes the conclusions.

II. MODEL AND SCALING ANALYSIS

A. Model

The 2D *XY* model is the simplest one that exhibits a KT phase transition. The Hamiltonian is written as

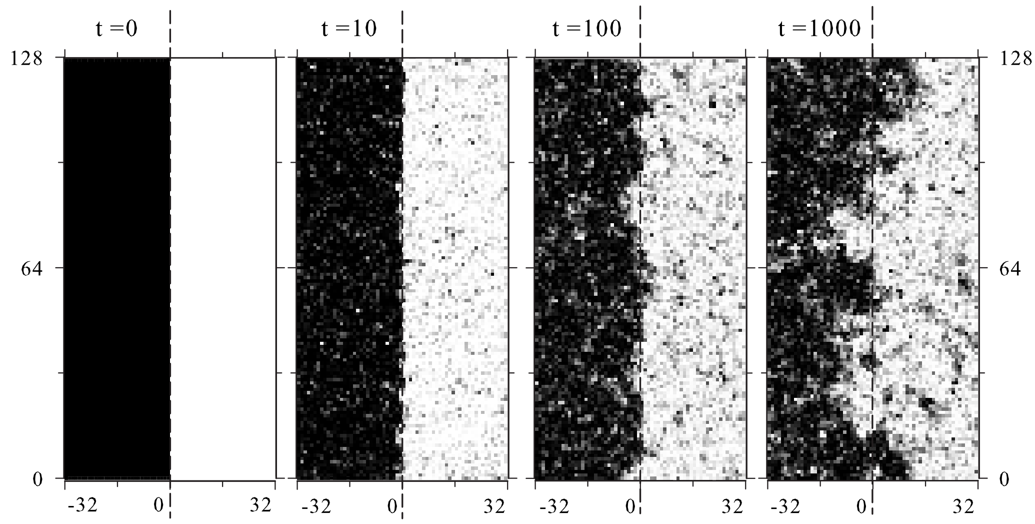

FIG. 1. Dynamic evolution of a domain wall for the 2D *XY* model at the temperature $T=0.89$, slightly below T_{KT}. The spin configuration of the domain interface is shown in a spatial window $[-32,32]$ at the time $t=0$, 10 100, and 1000 (from left to right). Black points denote $\vec{S}_i=(0,-1)$ and white points denote $\vec{S}_i=(0,1)$. The brightness of the grey points represents the y component of $\vec{S}_i$.

$$-\frac{1}{kT}H = K\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j, \tag{1}$$

where $\vec{S}_i=(S_{i,x},S_{i,y})$ is a planar unit vector at the site i of a lattice, the sum is over the nearest neighbors, and T is the temperature. For convenience, we take the notation $K=1/T$. In literature, the transition temperature T_{KT} is reported to be between 0.89 and 0.90 [55–57]. *Below T_{KT}, the system remains critical*. In this paper, we consider the standard Monte Carlo dynamics, which is believed to be in the same universality class of the Langevin dynamics. Following Refs. [11,51], we adopt the "heat-bath" algorithm of an one-spin flip, in which a trial move is accepted with the probability $1/[1+\exp(\Delta E/T)]$, where ΔE is the energy change associated with the move.

To study the dynamic relaxation of a domain wall, we first construct a *semiordered* initial state. Let us consider a rectangular lattice $2L\times L$, with the linear size $2L$ in the x direction and L in the y direction, and apply periodic boundary conditions in both directions. The semiordered state is a metastable state with a perfect domain wall, in which all spins $\vec{S}_i=(0,1)$ on the sublattice L^2 at the right side and $\vec{S}_i=(0,-1)$ on the sublattice L^2 at the left side. We set the x axis such that the domain wall between two domains is located at $x=0$. So the x coordinate of a lattice site is a half integer.

After preparing the semiordered initial state, we update the spins with the Monte Carlo algorithm at the temperature $T=0.89$, slightly below T_{KT}. Since no external magnetic field is applied, macroscopically the domain wall does not move. As time evolves, however, the domain wall fluctuates and creates bubbles. As a result, the domain wall *roughens*. Therefore we call it *a domain interface*. In Fig. 1, the dynamic evolution of the spin configuration around the domain wall is illustrated. Somewhat different from a standard growing interface, here the *bulk* evolves also in time. In analysis of the dynamic properties of the domain interface, this must be kept in mind.

Due to the semiordered initial state, the time evolution of the dynamic system is inhomogeneous in the x direction. Therefore we measure the magnetization and its second moment as functions of x and t,

$$\vec{M}^{(k)}(t,x) = \frac{1}{L^k}\left\langle\left[\sum_{y=1}^{L}\vec{S}_{xy}(t)\right]^k\right\rangle, \quad k=1,2. \tag{2}$$

Here $\vec{S}_{xy}(t)$ is the spin at time t on site (x,y), L is the lattice size, and $\langle\cdots\rangle$ represents the statistical average. From the symmetry of the semiordered initial state, the x component of the magnetization is zero, i.e., $\vec{M}^{(1)}(t,x)=(0,M^{(1)}(t,x))$. We denote $M(t,x)\equiv M^{(1)}(t,x)$ and $M^{(2)}(t,x)\equiv\vec{M}^{(2)}(t,x)$, then define a time-dependent Binder cumulant [8,48],

$$U(t,x) = M^{(2)}(t,x)/[M(t,x)]^2 - 1. \tag{3}$$

The Binder cumulant $U(t,x)$ describes the fluctuation in the y direction.

In order to directly characterize the growth of the domain interface and its fluctuation in the x direction, we introduce a height function and its second moment in the x direction,

$$\vec{h}^{(k)}(t) = \frac{1}{(L/2)^k}\left\langle\left[\sum_{x=1}^{L/2}\vec{S}_{xy}(t)\right]^k\right\rangle, \quad k=1,2. \tag{4}$$

Here $\langle\cdots\rangle$ represents not only the statistic average but also the average in the y direction. Again, we denote $\vec{h}^{(1)}(t)=(0,h(t))$ and $h^{(2)}(t)\equiv\vec{h}^{(2)}(t)$. Then the roughness function of the domain interface is defined as

$$\omega^2(t) = h^{(2)}(t) - h(t)h(t). \tag{5}$$

Except for the scaling dimension of the magnetization, the height function measures the thickness of the domain interface, while the roughness function represents its fluctuation.

To study the temporal correlation of the domain interface, we define the two-time correlation functions

$$A(t,t',x) = \frac{1}{L}\left\langle \sum_{y=1}^{L} \vec{S}_{xy}(t) \cdot \vec{S}_{xy}(t') \right\rangle, \quad t > t', \tag{6}$$

and

$$C(t,t',x) = A(t,t',x) - M(t,x)M(t',x). \tag{7}$$

$A(t,t',x)$ includes the contribution of the magnetization $M(t,x)$, and $C(t,t',x)$ describes the pure time correlation. From the definition, $C(t,t'=0,x)=0$.

B. Scaling analysis

In the critical regime, there are three spatial length scales in the dynamic system, i.e., the nonequilibrium spatial correlation length $\xi(t)$, the spatial coordinate x, and the lattice size L. In general, one may believe that $\xi(t)$ is isotropic in all spatial directions, because of the homogeneity of the interactions in the Hamiltonian. Therefore general scaling arguments lead to the scaling form of the magnetization and its second moment,

$$M^{(k)}(t,x,L) = \xi(t)^{-k\eta/2}\tilde{M}^{(k)}(\xi(t)/x, \xi(t)/L), \quad k=1,2. \tag{8}$$

Here η is the static exponent. On the right side of the equation, the overall factor $\xi(t)^{-k\eta/2}$ indicates the scaling dimension of $M^{(k)}$, and the scaling function $\tilde{M}^{(k)}(\xi(t)/x, \xi(t)/L)$ represents the scale invariance of the dynamic system. In general, the scaling form in Eq. (8) holds already in the *macroscopic* short-time regime, after a microscopic time scale t_{mic} [1,8].

For the magnetization, the scaling function $\tilde{M}(\xi(t)/x, \xi(t)/L)$ is independent of L in the thermodynamic limit $L \to \infty$. Then the scaling form is simplified to

$$M(t,x) = \xi(t)^{-\eta/2}\tilde{M}(\xi(t)/x). \tag{9}$$

The Binder cumulant, however, is different. Due to $\xi(t) \ll L$ in the short-time regime, the spatially correlating terms in the susceptibility $M^{(2)}(t,x) - M(t,x)^2$ can be neglected, and it leads to the finite-size behavior $U(t,x) \sim 1/L^{d-1}$ $(d=2)$ [8]. Together with Eqs. (8) and (9), one may derive the scaling form

$$U(t,x) = \xi(t)^{d-1}\tilde{U}(\xi(t)/x)/L^{d-1}, \quad d=2. \tag{10}$$

The Binder cumulant is interesting, for the static exponent η is *not* involved.

By the definition, the height function $h(t)$ is just the magnetization in the positive domain. Its behavior replies on the scaling function $\tilde{M}(\xi(t)/x)$. In fact, one may deduce a scaling form from Eq. (8),

$$h(t) = \xi(t)^{-\eta/2}\tilde{h}(\xi(t)/L). \tag{11}$$

Different from $M(t,x)$, one should not ignore the dependence of $h(t)$ on the lattice size L, for the scaling function $\tilde{h}(\xi(t)/L)$ just reflects the dynamic effect of the domain interface. Similar to Eq. (11), one may also assume the scaling form for the roughness function,

$$\omega^2(t) = \xi(t)^{-\eta}\tilde{\omega}^2(\xi(t)/L). \tag{12}$$

Different from a standard growing interface, $\omega^2(t)$ does not exhibit a power-law behavior of $\xi(t)$, for it includes fluctuations from the domain interface and the bulk.

For the time correlation functions, we may write down the dynamic scaling forms,

$$A(t,t',x) = \xi(t')^{-\eta}\tilde{A}(\xi(t)/\xi(t'), \xi(t')/x). \tag{13}$$

and

$$C(t,t',x) = \xi(t')^{-\eta}\tilde{C}(\xi(t)/\xi(t'), \xi(t')/x). \tag{14}$$

Since the scaling functions $\tilde{A}$ and $\tilde{C}$ depend on two scaling variables, the dynamic behavior described by $A(t,t',x)$ and $C(t,t',x)$ is relatively complicated.

The purpose of this paper is to clarify the scaling forms in Eqs. (8)–(14), and especially to determine the growth law of the nonequilibrium spatial correlation length $\xi(t)$ including corrections to scaling.

For the critical dynamics of a continuous phase transition, $\xi(t)$ usually grows by a power law $\xi(t) \sim t^{1/z}$, and z is the so-called dynamic exponent [1,8]. In shorter times, there may be corrections to scaling, typically in a power-law form

$$\xi(t) \sim t^{1/z}(1 + c/t^b). \tag{15}$$

For magnetic systems with a second-order phase transition such as the Ising model, the correction to scaling is usually rather weak, i.e., the correction exponent b is not so small. For magnetic systems with a KT phase transition, e.g., the 2D *XY* model, the correction to scaling remains weak in the dynamic relaxation starting from an *ordered* state, and the dynamic exponent is theoretically expected to be $z=2$ [11]. Due to the dynamic effect of the vortex-pair annihilation, however, the correction to scaling becomes strong in the dynamic relaxation starting from a *disordered* state, and essentially exhibits a logarithmic form [11,24,51,52,54],

$$\xi(t) \sim [t/(\ln t + c)]^{1/z}. \tag{16}$$

Theoretically, Eq. (16) is equivalent to Eq. (15) in the limit $b \to 0$. In numerical computations, $b \leqslant 0.1$ may already indicate a logarithmic correction to scaling. Numerically detecting a logarithmic correction to scaling is rather notorious, for it is negligible only in the limit $t \to \infty$. In this paper, we will show that *inside* the domain interface there also emerges a logarithmic correction to scaling, attributed to the vortex-pair annihilation.

III. MONTE CARLO SIMULATION

A. Magnetization

In Monte Carlo simulations, our main results are obtained with $L=256$ and $L=512$ at $T=0.89$, and the maximum updating time is $t_M = 10\,000$. Additional simulations with $L=1024$ are performed up to $t_M = 100\,000$, to detect the logarithmic corrections to scaling and to investigate possible finite-size effects. The total of samples for average is 10 000. The statistical errors are estimated by dividing the samples into two or three subgroups. If the fluctuation in the time direction is comparable with or larger than the statistical error, it will be taken into account. Theoretically, the scaling forms Eqs. (8)–(12) hold after a microscopic time scale t_{mic}. t_{mic} is not universal, and relies on microscopic details of the dynamic systems. In our simulations, t_{mic} is typically 100 or 200 Monte Carlo time steps.

The time evolution of the magnetization of the 2D *XY* model starting from the semiordered state is displayed in Fig. 2(a). According to Eq. (9), let us denote $s=\xi(t)/x$. For a sufficiently small s, e.g., $x=127.5$ and $t<10\,000$, $M(t,x)$ approaches the power-law decay *at bulk*. For a sufficiently large s, e.g., $x=0.5$ and $t>100$, $M(t,x)$ also appears to exhibit a power-law behavior, but decays *much faster* than at bulk. In other words, the scaling function $\tilde{M}(s)$ in Eq. (9) is characterized by

$$\tilde{M}(s) \sim \begin{cases} \text{const} & s \to 0 \\ s^{-\eta_0/2} & s \to \infty \end{cases} . \tag{17}$$

Outside the interface $s\to 0$, the dynamic relaxation of the magnetization is governed by the bulk exponent η, while inside the interface $s\to\infty$, it is controlled by both the interface exponent η_0 and the bulk exponent η. Outside the domain interface, the dynamic relaxation of the magnetization is the same as that with an ordered initial state, and the correction to scaling is weak. Assuming $\xi(t)\sim t^{1/z}$, one deduces $M(t,x)\sim t^{-\eta/2z}$ for $s\to 0$. In Fig. 2(a), the exponent $\eta/2z=0.0585(3)$ measured from the slope of the curve of $x=127.5$ is well consistent with $\eta=0.234(2)$ and $z=2$ reported in the literature [11,55]. Similarly, one derives $M(t,x)\sim t^{-(\eta+\eta_0)/2z}$ for $s\to\infty$. One measures $(\eta+\eta_0)/2z=0.516(4)$ from the slope of the curve of $x=0.5$, and then calculates $\eta_0/2=0.915(8)$ by taking $z=2$ as input.

For the domain interfaces of the 2D and 3D Ising models, the exponent $\eta_0/2=\beta_0/\nu$ is reported to be 0.998(5) and 1.001(6), respectively, very close to 1 [49]. It indicates that $M(t,x)$ is an analytic function of x. For the free and disordered surfaces, the exponent β_0/ν is naturally different from 1. *Does the exponent* $\eta_0/2$ *for the domain interface of the 2D XY model really deviate from 1?* Let us first examine the behavior $M(t,x)\sim x^{\eta_0/2}$ in the large-s regime. In Fig. 3(a), $M(t,x)$ is plotted as a function of x. For sufficiently large t and small x, $M(t,x)$ exhibits a power-law behavior. From the slope of the curve of $t=10240$, e.g., one measures $\eta_0/2=0.991(9)$, rather close to 1, and contradicting with $\eta_0/2=0.915(8)$ obtained from Fig. 2(a).

Our thought is that there exists a strong correction to scaling in the growth law of $\xi(t)$, described by Eqs. (15) or (16). For detecting this correction to scaling, we have performed extra simulations up to $t_M=100\,000$ with the lattice size $L=512$ and 1024. The curve of $x=0.5$ is displayed in the inset of Fig. 2(a). Careful analysis reveals that the curve can be fitted by the power-law correction in Eq. (15) with a *small* correction exponent b or equivalently by the logarithmic correction in Eq. (16). The fitting yields an exponent $\eta_0/2=1.01(2)$, consistent with $\eta_0/2=0.991(9)$ obtained from Fig. 3(a).

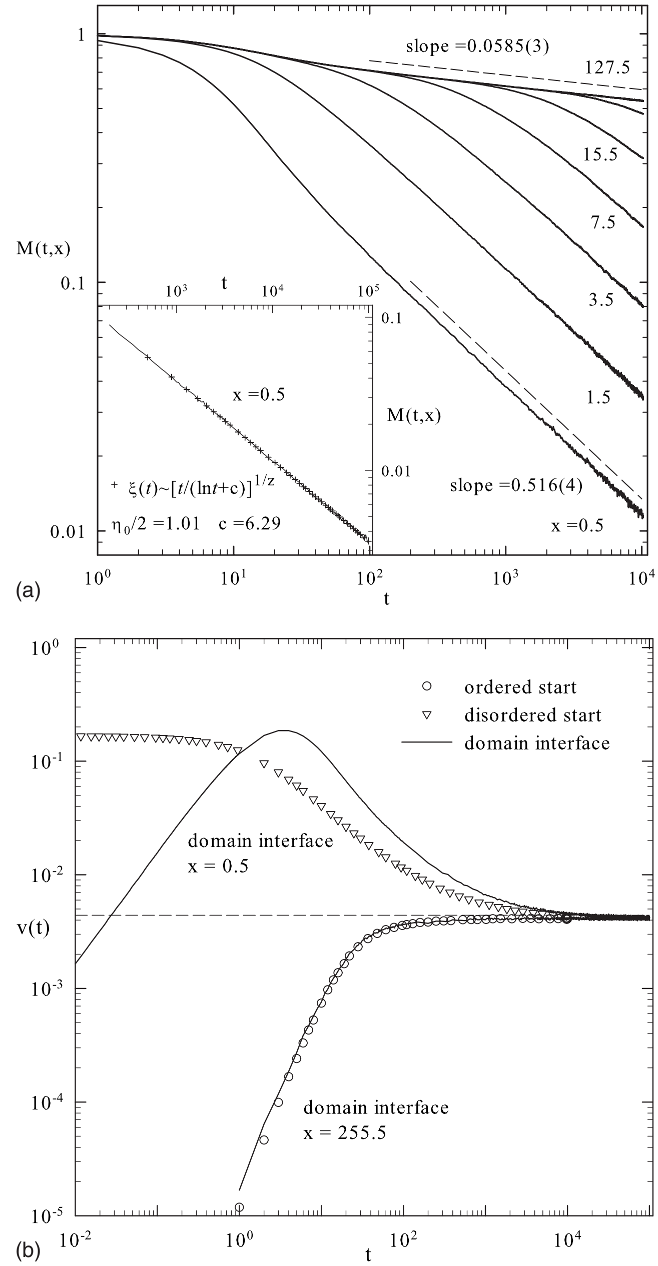

FIG. 2. (a) The time evolution of the magnetization of the 2D *XY* model starting from the semiordered state is displayed for different x with solid lines on a double-log scale. Dashed lines represent the power-law fits. In the inset, a fit with the logarithmic correction to scaling is shown for $x=0.5$ up to a longer time $t_M=100\,000$. (b) The vortex number evolves with time in the dynamic relaxation of a domain wall at $x=0.5$ and $x=255.5$, and in the dynamic relaxation starting from ordered and disordered states.

The logarithmic correction to scaling at the KT phase transition is believed to be induced by the vortex-pair annihilation. Therefore we measure the time evolution of the vortex number for different x, in comparison with those in the dynamic relaxation starting from ordered and disordered states. The results are shown in Fig. 2(b). The vortex number is defined as

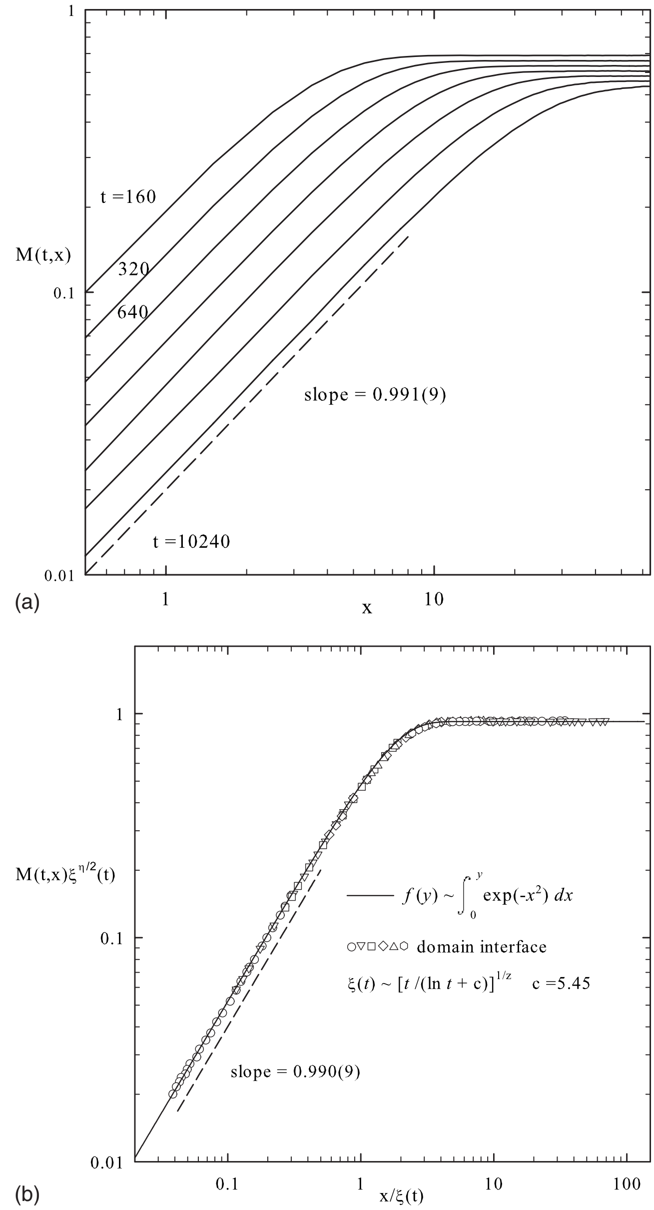

FIG. 3. (a) The magnetization of the 2D *XY* model starting from the semiordered state is plotted as a function of x for different t on a double-log scale. The dashed line shows a power-law fit. (b) The scaling function $\tilde{M}(\xi(t)/x)=M(t,x)\xi(t)^{\eta/2}$ is plotted as a function of $x/\xi(t)$ on a double-log scale. Inside the domain interface, logarithmic corrections to scaling are taken into account, and data collapse is observed for different x. The solid line represents the error function, and the dashed line shows a power-law fit..

$$v(t,x)=\langle|v_p|\rangle, \quad v_p=\sum_{p(x,y)}[\theta_i(t)-\theta_j(t)]/2\pi, \qquad (18)$$

where θ_i and θ_j denote the orientational angles of $\vec{S}_i$ and $\vec{S}_j$, $(\theta_i-\theta_j)$ are valued within $[-\pi,\pi]$, the sum is over the four links $\langle i,j\rangle$ of the clockwise plaquette at site (x,y), and $\langle\cdots\rangle$ represents both the statistical average and the average in the y direction. Outside the domain interface, e.g., at $x=255.5$, the vortex number is initially zero, then gradually increases with time, and finally reaches the steady value in equilibrium. This behavior is the same as that in the dynamic relaxation with an ordered initial state. Inside the domain interface, e.g., at $x=0.5$, the vortex number is also initially zero, but then rapidly jumps to a large value in a few Monte Carlo time steps, which even exceeds that in the dynamic relaxation with a disordered initial state. After reaching the maximum, the vortex number then decreases slowly and relaxes to the equilibrium. In other words, the dynamic effect of the vortex-pair annihilation in the dynamic relaxation of the domain wall is similar to or even stronger than that with a disordered initial state. Therefore it is not surprising that a logarithmic correction to scaling emerges.

To further verify the logarithmic correction to scaling in Eq. (16) and the scaling form in Eq. (9), we plot $\tilde{M}(s)=M(t,x)\xi(t)^{\eta/2}$ as a function of $1/s=x/\xi(t)$. According to Eq. (9), all data of different x should collapse onto the master curve $\tilde{M}(s)$. Inside the domain interface, the data collapse is rather sensitive to the corrections to scaling. Therefore our strategy is to determine the constant c in Eq. (16) by searching for the best data collapse, with $\eta=0.234$ and $z=2$ as input. This is shown in Fig. 3(b). Clearly, $\tilde{M}(s)\to$const when $s\to 0$, while $\tilde{M}(s)\to s^{-\eta_0/2}$ when $s\to\infty$. From the slope of the curve in large-s regime, we obtain $\eta_0/2=0.990(9)$, in agreement with the measurements in Figs. 2(a) and 3(a). We may also apply the power-law correction in Eq. (15) to the data collapse. It yields a correction exponent $b\approx 0.05$, consistent with the logarithmic correction in Eq. (16). Here we should mention that it is relatively difficult to observe the logarithmic correction simply from the time evolution of the magnetization at a fixed x in Fig. 2(a). However, the data collapse in Fig. 3(b) involves the dependence of the magnetization on both x and t, and therefore it is more efficient in detecting the logarithmic correction to scaling.

Additionally, we find that the scaling function $\tilde{M}(s)$ can be fitted to the error function $f(y)\sim\int^y \exp(-x^2)dx$, as shown in Fig. 3(b). In fact, this scaling function is rather robust, and it also applies to the domain interfaces of the 2D and 3D Ising models. It is a challenge to theoretically derive the scaling form and scaling function. Detailed results of this kind will be reported elsewhere.

B. Binder cumulant and height function

To clarify the scaling form of the Binder cumulant in Eq. (10), we plot $U(t,x)$ as a function of t for different x in Fig. 4(a). Outside the interface, e.g., $x=127.5$, a power-law behavior is observed, and the slope of the curve is 0.503(9). Assuming $\xi(t)\sim t^{1/z}$, one derives $U(t,x)\sim t^{(d-1)/z}$. Therefore we obtain $z=1.99(4)$, consistent with the theoretical value $z=2$. Inside the domain interface, e.g., $x=0.5$, $U(t,x)$ looks like exhibiting also a power-law behavior, and the slope of the curve is 1.382(10). Therefore it suggests

$$\tilde{U}(s)\sim\begin{cases}\text{const} & s\to 0\\ s^{d_0} & s\to\infty\end{cases}. \qquad (19)$$

Assuming $\xi(t)\sim t^{1/z}$, one derives $U(t)\sim t^{(d-1+d_0)/z}$. From $(d-1+d_0)/z=1.382(10)$, one calculates $d_0=1.764(20)$, significantly different from 2. For the 2D and 3D Ising models,

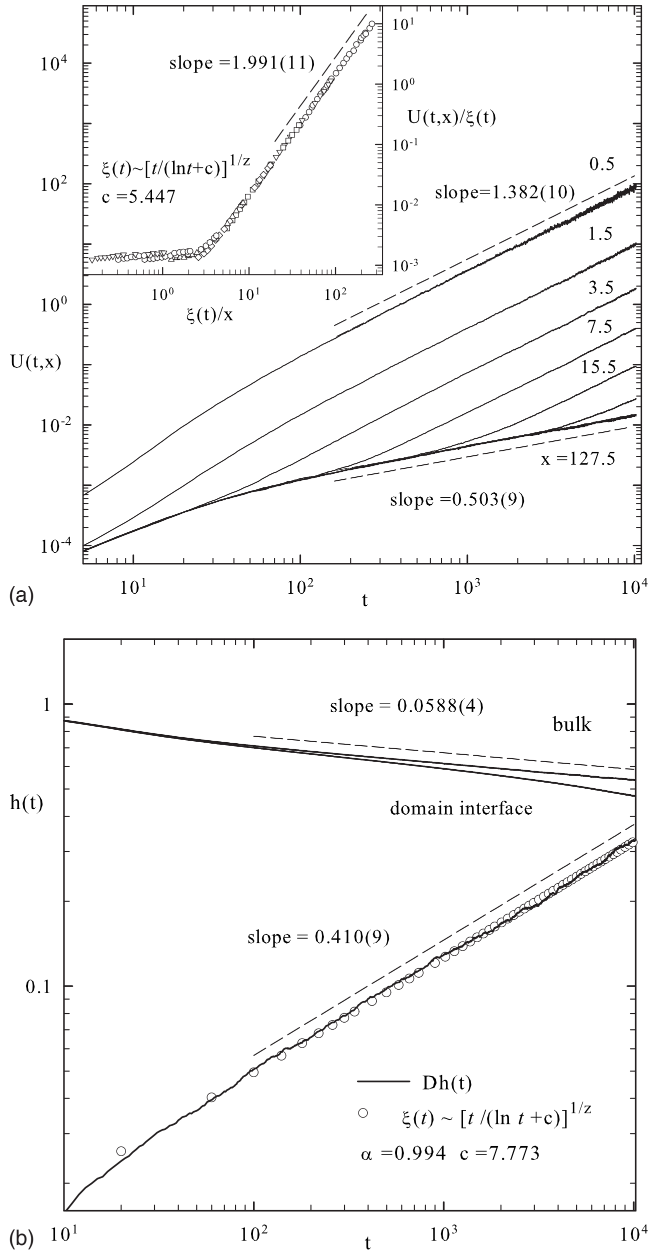

FIG. 4. (a) The Binder cumulant of the 2D *XY* model starting from the semiordered state is displayed for different x with solid lines on a double-log scale. Dashed lines represent the power-law fits. In the inset, the scaling function $\tilde{U}(\xi(t)/x)=U(t,x)/\xi(t)$ is plotted as a function of $\xi(t)/x$. Inside the domain interface, logarithmic corrections to scaling are taken into account. (b) Height functions of the domain interface, bulk, and pure domain interface are displayed on a double-log scale. For clarity, the curve of $Dh(t)$ is shifted up by a factor of 5. Dashed lines show the power-law fits, and circles indicate a fit with a logarithmic correction to scaling.

d_0 are estimated to be 2.00(2) and 2.01(2), respectively, very close to 2. The analysis above for the magnetization can be similarly applied to the Binder cumulant, and leads to the logarithmic correction to scaling in Eq. (16). The data collapse of $\tilde{U}(s)$ is shown in the inset of Fig. 4(a). From the slope of the curve in the large-s regime, one extracts d_0 =1.991(11), close to 2.

From the definition, our height function $h(t)$ just represents the thickness of the domain interface, and its scaling behavior is similar to that of $\omega(t)$ in Eq. (12). Therefore we simply focus on the height function $h(t)$. In Fig. 4(b), $h(t)$ of the domain interface decreases faster than a power law. Actually, the curve can be fitted by a double power law, e.g., $h(t)=c_0t^{b_0}-c_1t^{b_1}$. The conjecture is that the term $c_1t^{b_1}$ describes the pure interface, and $c_0t^{b_0}$ represents the magnetization of the bulk. Let us denote the magnetization of the bulk by $h_b(t)$, and assume that it can be simulated by the dynamic relaxation of the magnetization starting from an ordered state (i.e., without the domain wall). In Fig. 4(b), one observes that $h_b(t)$ decays by a power law, and the slope of the curve is 0.0588(4), consistent with that in the literature [11].

Now we define the *pure* height function for the domain interface by subtracting the contribution from the bulk,

$$Dh(t,L) = h_b(t) - h(t). \tag{20}$$

In Fig. 4(b), one observes that $Dh(t,L)$ looks like exhibiting a power-law behavior. The slope of the curves is estimated to be 0.410(9). In other words, one may assume

$$\tilde{h}(u) = \begin{cases} c & \text{bulk} \\ c + u^{\alpha} & \text{domain interface} \end{cases}, \tag{21}$$

with $u=\xi(t)/L$. Then one derives

$$Dh(t,L) = \xi(t)^{\alpha-\eta/2}/L^{\alpha}. \tag{22}$$

Taking into account that the scaling dimension of the magnetization is $-\eta/2$, α is nothing but the roughness exponent of the domain interface. Assuming $\xi(t)\sim t^{1/z}$, one calculates the exponent α=0.938(18) from $(\alpha-\eta/2)/z$=0.410(9). This value disagrees with α=1 estimated from $Dh(t,L)\sim L^{-\alpha}$. For the 2D and 3D Ising models, α is also very close to 1. Again this suggests the existence of the logarithmic correction to scaling in Eq. (16). After introducing the logarithmic correction to scaling, one obtains a refined value $(\alpha-\eta/2)/z$ =0.438(4), then derives α=0.994(8), close to 1.

C. Aging phenomena

The complication of the dynamic scaling forms of the time correlation functions in Eqs. (13) and (14) arises in two folds: there are two independent scaling variables and there exist strong corrections to scaling. According to the procedure in Refs. [24,48], one may *fix* the scaling variable s' $=\xi(t')/x$ at certain values, and plot $A(t,t',x)\xi(t')^{\eta}$ and $C(t,t',x)\xi(t')^{\eta}$ as functions of $\xi(t)/\xi(t')$. If the dynamic scaling forms hold, the data of different t' and x with a fixed s' should collapse. This actually is a kind of aging phenomena. If the correction to scaling is negligible, i.e., $\xi(t)\sim t^{1/z}$, the above performance is relatively straightforward. For example, it has been partially carried out for the 2D Ising models [48]. If the correction to scaling is strong, one should first determine the growth law $\xi(t)$. Otherwise, the correlation to scaling of $\xi(t)$ may mix with the deviation of the scaling function $\tilde{A}$ and $\tilde{C}$ from the power laws, and the analysis of the dynamic scaling behavior becomes complicated [24,53,54].

Direct measurements of $\xi(t)$ from the spatial correlation function is somewhat difficult. Fortunately, our $\xi(t)$ has been accurately determined from the time evolution of the magnetization in Sec. III A. Inside the domain interface, there is a strong logarithmic correction to scaling. Outside the domain interface, the correction to scaling is in a power-law form described by Eq. (15) with $b=1$, and practically rather weak, more or less negligible. In Fig. 5(a), the scaling function $A(t,t',x)\xi(t')^{\eta}$ is displayed for two typical values of s'. Obviously, data collapse is observed. The small value $s'=0.143$ shows the dynamic behavior of the bulk, i.e., outside the domain interface, while the "large" value $s'=0.603$ represents that inside the domain interface. Naively $s'=0.603$ looks not so large because of the logarithmic correction in Eq. (16). Actually the data are obtained with large t' and small x.

Let us denote $r=\xi(t)/\xi(t')$. Theoretically, in the large-r limit, the scaling function $\widetilde{A}(r,s')$ should approach $r^{-\eta/2}$ at the bulk, and $r^{-(\eta+\eta_0)/2}$ inside the domain interface [24,48]. In the medium-r regime, the scaling function $\widetilde{A}$ shows deviation from power laws. Careful analysis leads to the form

$$\widetilde{A}(r,s') = \begin{cases} r^{-\eta/2}(1+c'/r^2) & \text{bulk} \\ r^{-(\eta+\eta_0)/2}(1+c'/r) & \text{inside interface} \end{cases}. \tag{23}$$

Fitting the above ansatz to the data in Fig. 5(a), we estimate $\eta/2=0.115(6)$ and $(\eta+\eta_0)/2=1.119(5)$, consistent with $\eta/2=0.117(1)$ and $(\eta+\eta_0)/2=1.114(7)$ obtained from the magnetization in Sec. III A. At the bulk, the deviation of $\widetilde{A}(r,s')$ from a power law is described by the term c'/r^2, and it agrees with that in Ref. [24]. On the other hand, this term is reported to be c'/r for the dynamic relaxation with a disordered initial state [24]. Therefore $\widetilde{A}(r,s')$ inside the domain interface approaches the power-law limit in a similar form as that with a disordered initial state, but slower than that with an ordered initial state.

After subtracting the contribution of the magnetization from $A(t,t',x)$, $C(t,t',x)$ describes the pure time correlation. *At the bulk*, $C(t,t',x)\xi(t')^{\eta}$ does show data collapse. This is displayed with $s'=0.143$ in Fig. 5(b). Since $C(t,t',x)$ decays rapidly with time, the data in Fig. 5(b) are relatively fluctuating. But we could still observe a power-law tail. Theoretically, one could expect $\widetilde{C}(r,s')\sim r^{-(d+\eta/2)}$ at the bulk in the large-r regime. From the slope of the curve, one estimates $d+\eta/2=2.13(9)$, consistent with $d=2$ and $\eta/2=0.117(1)$ measured from the magnetization.

The dynamic behavior of $C(t,t',x)$ *inside the domain interface* is somewhat subtle. As shown in Fig. 5(b), the data of t' up to 320 with $s'=0.603$ do not collapse. However, all curves appear to be in a similar form. Possibly, an extra correction to scaling has not been under control. Anyway, we could fit the curves with the ansatz $\widetilde{C}(r,s')\sim r^{-\lambda}(1+c'/r)$, and extract $\lambda=0.862(8)$. This exponent is different from $\lambda=1.476(10)$ in the dynamic relaxation with a disordered initial state [11,24]. Further understanding of the dynamic behavior of $C(t,t',x)$ remains open.

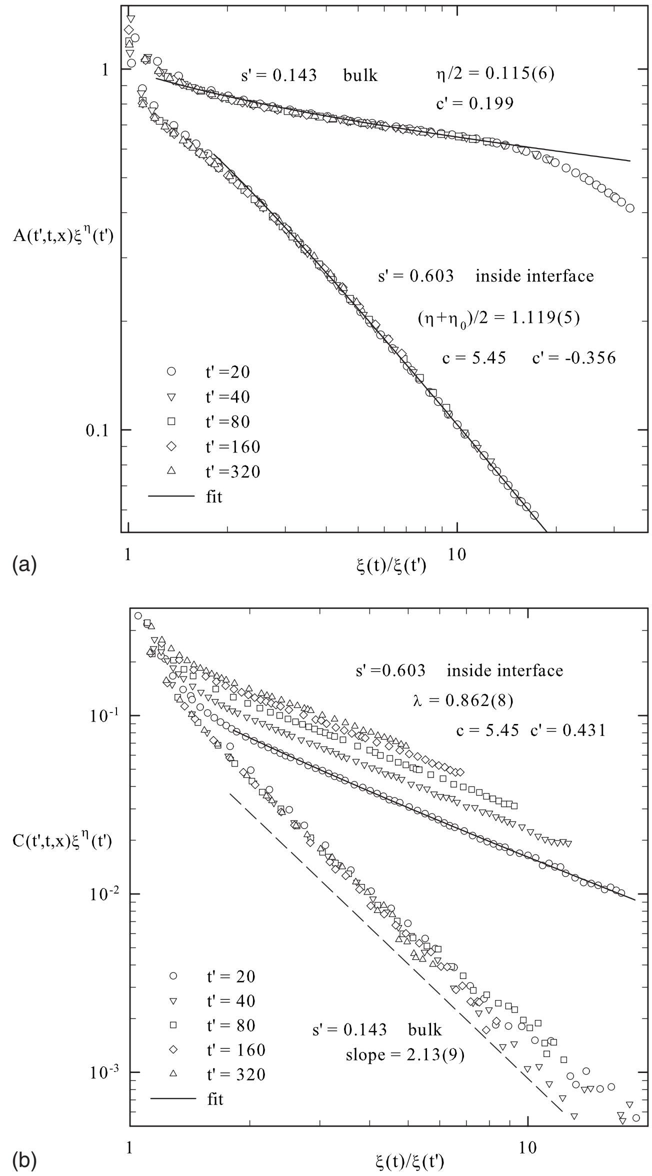

FIG. 5. (a) The scaling function $\widetilde{A}(\xi(t)/\xi(t'),s')=A(t,t',x)\xi(t')^{\eta}$ with a fixed $s'=\xi(t')/x$ is plotted as a function of $\xi(t)/\xi(t')$ on a double-log scale. Inside the domain interface, logarithmic corrections to scaling are taken into account. Solid lines represent the fits with Eq. (23). (b) The scaling function $\widetilde{C}(\xi(t)/\xi(t'),s')=C(t,t',x)\xi(t')^{\eta}$ with a fixed $s'=\xi(t')/x$ is plotted as a function of $\xi(t)/\xi(t')$ on a double-log scale. Inside the domain interface, logarithmic corrections to scaling are taken into account, but data collapse is not observed. The dashed line shows a power-law fit, and the solid line represents a fit with $\widetilde{C}(r,s')\sim r^{-\lambda}(1+c'/r)$.

IV. CONCLUSION

In summary, we have simulated the dynamic relaxation of domain walls at the KT phase transition of the 2D *XY* model with Monte Carlo methods. The dynamic scaling behavior of the magnetization, Binder cumulant, height function, and two-time correlation function is carefully analyzed, and a domain-wall roughening process is observed. A strong logarithmic correction to scaling is detected inside the domain interface. After taking into account the logarithmic corrections to scaling, the dynamic scaling forms of different physical observables are accurately verified, and the naive measurements of the exponents $\eta_0=0.915(8)$, $d_0=1.764(20)$, and $\alpha=0.938(18)$ are refined to be $\eta_0=0.997(7)$, $d_0=1.991(11)$, and $\alpha=0.994(8)$. We have also understood a large part of the aging phenomena. A similar dynamic approach may be applied to the phase transitions of domain walls driven by external magnetic fields at the zero or low temperature. The re-orientation dynamics of the magnetization around the domain interface is also interesting.

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